

ISO/TS 28037:2010-09 (E)

Determination and use of straight-line calibration functions

Contents	Page
Foreword	v
Introduction	vi
1 Scope	1
2 Normative references	1
3 Terms and definitions	1
4 Conventions and notation	4
5 Principles of straight-line calibration	5
5.1 General	5
5.2 Inputs to determining the calibration function	5
5.2.1 Measurement data	5
5.2.2 Associated uncertainties and covariances	6
5.3 Determining the calibration function	6
5.4 Numerical treatment	7
5.5 Uncertainties and covariance associated with the calibration function parameters	7
5.6 Validation of the model	8
5.7 Use of the calibration function	8
5.8 Determining the ordinary least squares best-fit straight line to data	9
6 Model for uncertainties associated with the y_i	9
6.1 General	9
6.2 Calibration parameter estimates and associated standard uncertainties and covariance ..	10
6.3 Validation of the model	11
6.4 Organization of the calculations	11
7 Model for uncertainties associated with the x_i and the y_i	17
7.1 General	17
7.2 Calibration parameter estimates and associated standard uncertainties and covariance ..	18
7.3 Validation of the model	20
7.4 Organization of the calculations	20
8 Model for uncertainties associated with the x_i and the y_i and covariances associated with the pairs (x_i, y_i)	24
8.1 General	24
8.2 Calibration parameter estimates and associated standard uncertainties and covariance ..	24
9 Model for uncertainties and covariances associated with the y_i	25
9.1 General	25
9.2 Calibration parameter estimates and associated standard uncertainties and covariance ..	25
9.3 Validation of the model	27
9.4 Organization of the calculations	27
10 Model for uncertainties and covariances associated with the x_i and the y_i	31
10.1 General	31
10.2 Calibration parameter estimates and associated standard uncertainties and covariance ..	31
10.3 Validation of the model	34

11	Use of the calibration function	37
11.1	Prediction	37
11.2	Forward evaluation	39
A Annexes A (informative) Matrix operations	41	
A.1	General	41
A.2	Elementary operations	41
A.2.1	Matrix-vector multiplication	41
A.2.2	Matrix-matrix multiplication	41
A.2.3	Matrix transpose	41
A.2.4	Identity matrix	41
A.2.5	Inverse of a square matrix	41
A.3	Elementary definitions	41
A.3.1	Symmetric matrix	42
A.3.2	Invertible matrix	42
A.3.3	Lower-triangular and upper-triangular matrix	42
A.3.4	Orthogonal matrix	42
A.4	Cholesky factorization	42
A.4.1	Cholesky factorization algorithms	42
A.4.2	Interpretation of the Cholesky factorization of a covariance matrix	43
A.4.3	Solution of a lower-triangular system	43
A.4.4	Solution of an upper-triangular system	44
A.5	Orthogonal factorization	44
A.5.1	QR factorization	45
A.5.2	RQ factorization	45
B (informative) Application of the Gauss-Newton algorithm to generalized distance regression	46	
C (informative) Orthogonal factorization approach to solving the generalized Gauss-Markov prob- lem	48	
C.1	General	48
C.2	Calibration parameter estimates and associated standard uncertainties and covariance	48
C.3	Validation of the model	49
D (informative) Provision of uncertainties and covariances associated with the measured x- and y-values	52	
D.1	General	52
D.2	Response data 1	52
D.2.1	General	52
D.2.2	Measurement model for uncertainties and covariances associated with the y_i	52
D.3	Response data 2	53
D.4	Stimulus data 1	53
D.5	Stimulus data 2	54
D.6	Stimulus and response data	55
E (informative) Uncertainties known up to a scale factor	56	
F (informative) Software implementation of described algorithms	60	
G (informative) Glossary of principal symbols	61	
Bibliography	63	