

# ISO/IEC 10967-2:2001-08 (E)

## Information technology - Language independent arithmetic - Part 2: Elementary numerical functions

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