ISO/IEC 40314:2016-03 (E)

Information technology - Mathematical Markup Language (MathML) Version 3.0; 2nd Edition

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