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Uncertainty of measurement_- Part_3: Guide to the expression of uncertainty in measurement (GUM:1995)_- Supplement_1: Propagation of distributions using a Monte Carlo method

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