

# ISO 10303-50:2002-05 (E)

## Industrial automation systems and integration - Product data representation and exchange - Part 50: Integrated generic resource: Mathematical constructs

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