ISO/TR 18532:2009-04 (E)

Guidance on the application of statistical methods to quality and to industrial standardization

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