

# ISO/IEC 40314:2016-03 (E)

## Information technology - Mathematical Markup Language (MathML) Version 3.0; 2nd Edition

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Contents	Page
<b>1 Introduction</b>	<b>9</b>
1.1 Mathematics and its Notation . . . . .	9
1.2 Origins and Goals . . . . .	10
1.2.1 Design Goals of MathML . . . . .	10
1.3 Overview . . . . .	11
1.4 A First Example . . . . .	11
<b>2 MathML Fundamentals</b>	<b>14</b>
2.1 MathML Syntax and Grammar . . . . .	14
2.1.1 General Considerations . . . . .	14
2.1.2 MathML and Namespaces . . . . .	14
2.1.3 Children versus Arguments . . . . .	15
2.1.4 MathML and Rendering . . . . .	15
2.1.5 MathML Attribute Values . . . . .	15
2.1.6 Attributes Shared by all MathML Elements . . . . .	20
2.1.7 Collapsing Whitespace in Input . . . . .	21
2.2 The Top-Level <math> Element . . . . .	22
2.2.1 Attributes . . . . .	22
2.2.2 Deprecated Attributes . . . . .	24
2.3 Conformance . . . . .	24
2.3.1 MathML Conformance . . . . .	24
2.3.2 Handling of Errors . . . . .	27
2.3.3 Attributes for unspecified data . . . . .	27
<b>3 Presentation Markup</b>	<b>28</b>
3.1 Introduction . . . . .	28
3.1.1 What Presentation Elements Represent . . . . .	28
3.1.2 Terminology Used In This Chapter . . . . .	29
3.1.3 Required Arguments . . . . .	30
3.1.4 Elements with Special Behaviors . . . . .	31
3.1.5 Directionality . . . . .	32
3.1.6 Displaystyle and Scriptlevel . . . . .	33
3.1.7 Linebreaking of Expressions . . . . .	34
3.1.8 Warning about fine-tuning of presentation . . . . .	35
3.1.9 Summary of Presentation Elements . . . . .	37
3.1.10 Mathematics style attributes common to presentation elements . . . . .	38
3.2 Token Elements . . . . .	38
3.2.1 Token Element Content Characters, <mglyph/> . . . . .	39
3.2.2 Mathematics style attributes common to token elements . . . . .	41
3.2.3 Identifier <mi> . . . . .	45

3.2.4	Number <code>&lt;mn&gt;</code> . . . . .	46
3.2.5	Operator, Fence, Separator or Accent <code>&lt;mo&gt;</code> . . . . .	47
3.2.6	Text <code>&lt;mtext&gt;</code> . . . . .	60
3.2.7	Space <code>&lt;mspace/&gt;</code> . . . . .	62
3.2.8	String Literal <code>&lt;ms&gt;</code> . . . . .	64
3.3	General Layout Schemata . . . . .	64
3.3.1	Horizontally Group Sub-Expressions <code>&lt;mrow&gt;</code> . . . . .	64
3.3.2	Fractions <code>&lt;mfrac&gt;</code> . . . . .	67
3.3.3	Radicals <code>&lt;msqrt&gt;</code> , <code>&lt;mroot&gt;</code> . . . . .	69
3.3.4	Style Change <code>&lt;mstyle&gt;</code> . . . . .	69
3.3.5	Error Message <code>&lt;merror&gt;</code> . . . . .	72
3.3.6	Adjust Space Around Content <code>&lt;mpadded&gt;</code> . . . . .	73
3.3.7	Making Sub-Expressions Invisible <code>&lt;mphantom&gt;</code> . . . . .	78
3.3.8	Expression Inside Pair of Fences <code>&lt;mfenced&gt;</code> . . . . .	80
3.3.9	Enclose Expression Inside Notation <code>&lt;menclase&gt;</code> . . . . .	83
3.4	Script and Limit Schemata . . . . .	85
3.4.1	Subscript <code>&lt;msub&gt;</code> . . . . .	86
3.4.2	Superscript <code>&lt;msup&gt;</code> . . . . .	87
3.4.3	Subscript-superscript Pair <code>&lt;msubsup&gt;</code> . . . . .	87
3.4.4	Underscript <code>&lt;munder&gt;</code> . . . . .	88
3.4.5	Overscript <code>&lt;mover&gt;</code> . . . . .	89
3.4.6	Underscript-overscript Pair <code>&lt;munderover&gt;</code> . . . . .	91
3.4.7	Prescripts and Tensor Indices <code>&lt;mmultiscripts&gt;</code> , <code>&lt;mprescripts/&gt;</code> , <code>&lt;none/&gt;</code> . . . . .	93
3.5	Tabular Math . . . . .	95
3.5.1	Table or Matrix <code>&lt;mtable&gt;</code> . . . . .	95
3.5.2	Row in Table or Matrix <code>&lt;mtr&gt;</code> . . . . .	99
3.5.3	Labeled Row in Table or Matrix <code>&lt;mlabeledtr&gt;</code> . . . . .	99
3.5.4	Entry in Table or Matrix <code>&lt;mtd&gt;</code> . . . . .	101
3.5.5	Alignment Markers <code>&lt;maligngroup/&gt;</code> , <code>&lt;malignmark/&gt;</code> . . . . .	101
3.6	Elementary Math . . . . .	110
3.6.1	Stacks of Characters <code>&lt;mstack&gt;</code> . . . . .	111
3.6.2	Long Division <code>&lt;mlongdiv&gt;</code> . . . . .	113
3.6.3	Group Rows with Similiar Positions <code>&lt;msgroup&gt;</code> . . . . .	114
3.6.4	Rows in Elementary Math <code>&lt;msrow&gt;</code> . . . . .	115
3.6.5	Carries, Borrows, and Crossouts <code>&lt;mscarries&gt;</code> . . . . .	115
3.6.6	A Single Carry <code>&lt;mscarry&gt;</code> . . . . .	116
3.6.7	Horizontal Line <code>&lt;msline/&gt;</code> . . . . .	117
3.6.8	Elementary Math Examples . . . . .	118
3.7	Enlivening Expressions . . . . .	124
3.7.1	Bind Action to Sub-Expression <code>&lt;maction&gt;</code> . . . . .	124
3.8	Semantics and Presentation . . . . .	126
<b>4</b>	<b>Content Markup</b> . . . . .	<b>127</b>
4.1	Introduction . . . . .	127
4.1.1	The Intent of Content Markup . . . . .	127
4.1.2	The Structure and Scope of Content MathML Expressions . . . . .	128
4.1.3	Strict Content MathML . . . . .	128
4.1.4	Content Dictionaries . . . . .	129
4.1.5	Content MathML Concepts . . . . .	130
4.2	Content MathML Elements Encoding Expression Structure . . . . .	131

4.2.1	Numbers <code>&lt;cn&gt;</code> . . . . .	132
4.2.2	Content Identifiers <code>&lt;ci&gt;</code> . . . . .	138
4.2.3	Content Symbols <code>&lt;csymbol&gt;</code> . . . . .	140
4.2.4	String Literals <code>&lt;cs&gt;</code> . . . . .	142
4.2.5	Function Application <code>&lt;apply&gt;</code> . . . . .	143
4.2.6	Bindings and Bound Variables <code>&lt;bind&gt;</code> and <code>&lt;bvar&gt;</code> . . . . .	146
4.2.7	Structure Sharing <code>&lt;share&gt;</code> . . . . .	148
4.2.8	Attribution via <code>semantics</code> . . . . .	150
4.2.9	Error Markup <code>&lt;error&gt;</code> . . . . .	151
4.2.10	Encoded Bytes <code>&lt;cbytes&gt;</code> . . . . .	152
4.3	Content MathML for Specific Structures . . . . .	152
4.3.1	Container Markup . . . . .	153
4.3.2	Bindings with <code>&lt;apply&gt;</code> . . . . .	154
4.3.3	Qualifiers . . . . .	156
4.3.4	Operator Classes . . . . .	162
4.3.5	Non-strict Attributes . . . . .	169
4.4	Content MathML for Specific Operators and Constants . . . . .	170
4.4.1	Functions and Inverses . . . . .	170
4.4.2	Arithmetic, Algebra and Logic . . . . .	180
4.4.3	Relations . . . . .	200
4.4.4	Calculus and Vector Calculus . . . . .	205
4.4.5	Theory of Sets . . . . .	224
4.4.6	Sequences and Series . . . . .	233
4.4.7	Elementary classical functions . . . . .	243
4.4.8	Statistics . . . . .	247
4.4.9	Linear Algebra . . . . .	253
4.4.10	Constant and Symbol Elements . . . . .	260
4.5	Deprecated Content Elements . . . . .	268
4.5.1	Declare <code>&lt;declare&gt;</code> . . . . .	268
4.5.2	Relation <code>&lt;reln&gt;</code> . . . . .	268
4.5.3	Relation <code>&lt;fn&gt;</code> . . . . .	268
4.6	The Strict Content MathML Transformation . . . . .	268
<b>5</b>	<b>Mixing Markup Languages for Mathematical Expressions</b> . . . . .	<b>272</b>
5.1	Annotation Framework . . . . .	272
5.1.1	Annotation elements . . . . .	272
5.1.2	Annotation keys . . . . .	273
5.1.3	Alternate representations . . . . .	274
5.1.4	Content equivalents . . . . .	275
5.1.5	Annotation references . . . . .	276
5.2	Elements for Semantic Annotations . . . . .	276
5.2.1	The <code>&lt;semantics&gt;</code> element . . . . .	276
5.2.2	The <code>&lt;annotation&gt;</code> element . . . . .	277
5.2.3	The <code>&lt;annotation-xml&gt;</code> element . . . . .	278
5.3	Combining Presentation and Content Markup . . . . .	281
5.3.1	Presentation Markup in Content Markup . . . . .	281
5.3.2	Content Markup in Presentation Markup . . . . .	282
5.4	Parallel Markup . . . . .	282
5.4.1	Top-level Parallel Markup . . . . .	282
5.4.2	Parallel Markup via Cross-References . . . . .	283

<b>6</b>	<b>Interactions with the Host Environment</b>	<b>286</b>
6.1	Introduction . . . . .	286
6.2	Invoking MathML Processors . . . . .	286
6.2.1	Recognizing MathML in XML . . . . .	286
6.2.2	Recognizing MathML in HTML . . . . .	287
6.2.3	Resource Types for MathML Documents . . . . .	287
6.2.4	Names of MathML Encodings . . . . .	287
6.3	Transferring MathML . . . . .	288
6.3.1	Basic Transfer Flavor Names and Contents . . . . .	288
6.3.2	Recommended Behaviors when Transferring . . . . .	289
6.3.3	Discussion . . . . .	289
6.3.4	Examples . . . . .	290
6.4	Combining MathML and Other Formats . . . . .	292
6.4.1	Mixing MathML and XHTML . . . . .	294
6.4.2	Mixing MathML and non-XML contexts . . . . .	294
6.4.3	Mixing MathML and HTML . . . . .	294
6.4.4	Linking . . . . .	295
6.4.5	MathML and Graphical Markup . . . . .	296
6.5	Using CSS with MathML . . . . .	297
6.5.1	Order of processing attributes versus style sheets . . . . .	298
<b>7</b>	<b>Characters, Entities and Fonts</b>	<b>299</b>
7.1	Introduction . . . . .	299
7.2	Unicode Character Data . . . . .	299
7.3	Entity Declarations . . . . .	300
7.4	Special Characters Not in Unicode . . . . .	300
7.5	Mathematical Alphanumeric Symbols . . . . .	300
7.6	Non-Marking Characters . . . . .	303
7.7	Anomalous Mathematical Characters . . . . .	303
7.7.1	Keyboard Characters . . . . .	303
7.7.2	Pseudo-scripts . . . . .	304
7.7.3	Combining Characters . . . . .	306
<b>A</b>	<b>Parsing MathML</b>	<b>308</b>
A.1	Use of MathML as Well-Formed XML . . . . .	308
A.2	Using the RelaxNG Schema for MathML3 . . . . .	308
A.2.1	Full MathML . . . . .	309
A.2.2	Elements Common to Presentation and Content MathML . . . . .	309
A.2.3	The Grammar for Presentation MathML . . . . .	311
A.2.4	The Grammar for Strict Content MathML3 . . . . .	323
A.2.5	The Grammar for Content MathML . . . . .	325
A.2.6	MathML as a module in a RelaxNG Schema . . . . .	332
A.3	Using the MathML DTD . . . . .	333
A.3.1	Document Validation Issues . . . . .	333
A.3.2	Attribute values in the MathML DTD . . . . .	333
A.3.3	DOCTYPE declaration for MathML . . . . .	334
A.4	Using the MathML XML Schema . . . . .	334
A.4.1	Associating the MathML schema with MathML fragments . . . . .	334
A.5	Parsing MathML in XHTML . . . . .	334
A.6	Parsing MathML in HTML . . . . .	334

<b>B</b>	<b>Media Types Registrations</b>	<b>335</b>
B.1	Selection of Media Types for MathML Instances . . . . .	335
B.2	Media type for Generic MathML . . . . .	336
B.3	Media type for Presentation MathML . . . . .	337
B.4	Media type for Content MathML . . . . .	338
<b>C</b>	<b>Operator Dictionary (Non-Normative)</b>	<b>340</b>
C.1	Indexing of the operator dictionary . . . . .	340
C.2	Format of operator dictionary entries . . . . .	340
C.3	Notes on <code>lspace</code> and <code>rspace</code> attributes . . . . .	341
C.4	Operator dictionary entries . . . . .	341
<b>D</b>	<b>Glossary (Non-Normative)</b>	<b>379</b>
<b>E</b>	<b>Working Group Membership and Acknowledgments (Non-Normative)</b>	<b>383</b>
E.1	The Math Working Group Membership . . . . .	383
E.2	Acknowledgments . . . . .	386
<b>F</b>	<b>Changes (Non-Normative)</b>	<b>387</b>
F.1	Changes between MathML 3.0 First Edition and Second Edition . . . . .	387
F.2	Changes between MathML 2.0 Second Edition and MathML 3.0 . . . . .	390
<b>G</b>	<b>Normative References</b>	<b>391</b>
<b>H</b>	<b>References (Non-Normative)</b>	<b>393</b>
<b>I</b>	<b>Index (Non-Normative)</b>	<b>395</b>
I.1	MathML Elements . . . . .	395
I.2	MathML Attributes . . . . .	400