

ISO/IEC Guide 98-3 Supplement 2:2011-11 (E)

Extension to any number of output quantities

Contents	Page
Foreword	v
Introduction	vi
1 Scope	1
2 Normative references	2
3 Terms and definitions	2
4 Conventions and notation	8
5 Basic principles	10
5.1 General	10
5.2 Main stages of uncertainty evaluation	10
5.3 Probability density functions for the input quantities	11
5.3.1 General	11
5.3.2 Multivariate <i>t</i> -distribution	11
5.3.3 Construction of multivariate probability density functions	12
5.4 Propagation of distributions	12
5.5 Obtaining summary information	13
5.6 Implementations of the propagation of distributions	13
6 GUM uncertainty framework	14
6.1 General	14
6.2 Propagation of uncertainty for explicit multivariate measurement models	15
6.2.1 General	15
6.2.2 Examples	15
6.3 Propagation of uncertainty for implicit multivariate measurement models	17
6.3.1 General	17
6.3.2 Examples	17
6.4 Propagation of uncertainty for models involving complex quantities	19
6.5 Coverage region for a vector output quantity	19
6.5.1 General	19
6.5.2 Bivariate case	20
6.5.3 Multivariate case	21
6.5.4 Coverage region for the expectation of a multivariate Gaussian distribution	22
7 Monte Carlo method	23
7.1 General	23
7.2 Number of Monte Carlo trials	25
7.3 Making draws from probability distributions	25
7.4 Evaluation of the vector output quantity	27
7.5 Discrete representation of the distribution function for the output quantity	27
7.6 Estimate of the output quantity and the associated covariance matrix	27
7.7 Coverage region for a vector output quantity	28
7.7.1 General	28
7.7.2 Hyper-ellipsoidal coverage region	28
7.7.3 Hyper-rectangular coverage region	29
7.7.4 Smallest coverage region	30

7.8	Adaptive Monte Carlo procedure	31
7.8.1	General	31
7.8.2	Numerical tolerance associated with a numerical value	32
7.8.3	Adaptive procedure	33
8	Validation of the GUM uncertainty framework using a Monte Carlo method	34
9	Examples	35
9.1	Illustrations of aspects of this Supplement	35
9.2	Additive measurement model	36
9.2.1	Formulation	36
9.2.2	Propagation and summarizing: case 1	36
9.2.3	Propagation and summarizing: case 2	38
9.2.4	Propagation and summarizing: case 3	41
9.3	Co-ordinate system transformation	41
9.3.1	Formulation	41
9.3.2	Propagation and summarizing: zero covariance	44
9.3.3	Propagation and summarizing: non-zero covariance	45
9.3.4	Discussion	49
9.4	Simultaneous measurement of resistance and reactance	52
9.4.1	Formulation	52
9.4.2	Propagation and summarizing	52
9.5	Measurement of Celsius temperature using a resistance thermometer	55
9.5.1	General	55
9.5.2	Measurement of a single Celsius temperature	55
9.5.3	Measurement of several Celsius temperatures	56

Annexes

A	(informative) Derivatives of complex multivariate measurement functions	59
B	(informative) Evaluation of sensitivity coefficients and covariance matrix for multivariate measurement models	61
C	(informative) Co-ordinate system transformation	62
C.1	General	62
C.2	Analytical solution for a special case	62
C.3	Application of the GUM uncertainty framework	64
D	(informative) Glossary of principal symbols	65
	Bibliography	69
	Alphabetical index	71