

# ISO 21771-1:2024-10 (E)

## Cylindrical involute gears and gear pairs - Part 1: Concepts and geometry

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